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Paper

Optimization of texture design parameters for friction reduction based on design of experiments and finite element fluid analysis

Doan Quang THUAN[1], Takahiko KURAHASHI[2], Hiroaki ARATA[3]
and Tetsuro IYAMA[4]

[1]Mechanical Engineering, Master's Program, Graduate School of Engineering,
Nagaoka University of Technology
[2]Department of Mechanical Engineering, Institute of GIGAKU, Nagaoka University of Technology
[3]Mechanical Engineering, Master's Program, Graduate School of Engineering,
Nagaoka University of Technology (Current affiliation: Takasago Thermal Engineering Co., Ltd.)
[4]Department of Mechanical Engineering, National Institute of Technology, Nagaoka College

In this study, we present a new practical optimum design method that consists of two steps: finite element analysis (FEA) and design of experiments. The design of experiments is used to generate approximate evaluation functions for controlling the behavior depending on the changes in the design variables of an object structure by finite element analyses. Here, we used a design of experiments to determine the optimal combination of design parameters in the texture analysis for friction coefficient reduction. The effects of the design variables can be calculated based on an orthogonal array of design variable combinations. The approximate evaluation functions were then generated by these effects based on the analysis of variance. The proposed method was found to be an effective and powerful tool for the optimum design of various practical design problems.

Key Words: *Textures, Design Parameters, Friction Reduction, Design of Experiments Method*

1. Introduction

Against the backdrop of the growing awareness of environmental issues and the strengthening of the international competitiveness of the manufacturing industry, there is a need to improve the thermal efficiency and reliability of transportation and industrial machinery and increase their functionality and added value. The improvement of tribological properties is a fundamental and core technology for solving these problems and core technology and is required to achieve more advanced technological requirements. In the development of tribological properties, it is essential to reduce costs, improve performance, and use materials, lubricants, and manufacturing processes that have lower environmental impact. Surface modification is a reasonable material-creation technology that adds the properties required for the surface independent of the interior and improves the performance of the material.

Surface texturing is one of the most prominent surface modification technologies that creates "texture" by machining grooves or holes in sliding surfaces. By applying these textures to sliding surfaces that are adequately lubricated with oil, friction can be reduced. Surface texturing is a simple and essential surface treatment process. Therefore, various numerical models have been employed in many studies to determine the optimal texturing parameters (shape, size, and distribution) for optimal performance enhancement in terms of friction and wear. However, the large number of parameters and complexity of their combinations render determining the optimum texture a challenging task, leading to contrary conclusions.

Hence, in this study, we used a design of experiments[1)] to determine the optimal combination of design parameters in the texture analysis for friction coefficient reduction. FreeFEM++ was used in the finite-element fluid analysis to determine the coefficient of friction[2)].

2. Formulation

This study assumes fluid-lubricated sliding surfaces in which fluid is present between two surfaces that are in relative motion without contact. The computational model is shown in Fig. 1. The whole domain is defined as Ω and its boundary as Γ. The governing equation is calculated for domain Ω, and the boundary conditions are given in Γ.

The friction coefficient μ is obtained by dividing the frictional force F by the load, W as shown in Eq. (1) [3), 4)]

$$\mu = \frac{F}{W} = \frac{\int_{\Omega} \left(\frac{\eta U}{h} + \frac{h}{2}\frac{\partial p}{\partial x}\right) d\Omega}{\int_{\Omega} p \, d\Omega} \tag{1}$$

where η and U are the viscosity and velocity in the x-direction of the friction surface, respectively, and are constants. h and p are the oil film thickness and pressure, respectively, and are variables.

The Reynolds equation (Eq. (2)) was used as the governing equation based on the Reynolds assumption, as follows:

[1] The fluid flow is laminar.

[2] The fluid is incompressible.

[3] Viscosity is constant.

[4] Gravity and inertial forces are negligible compared to viscous forces.

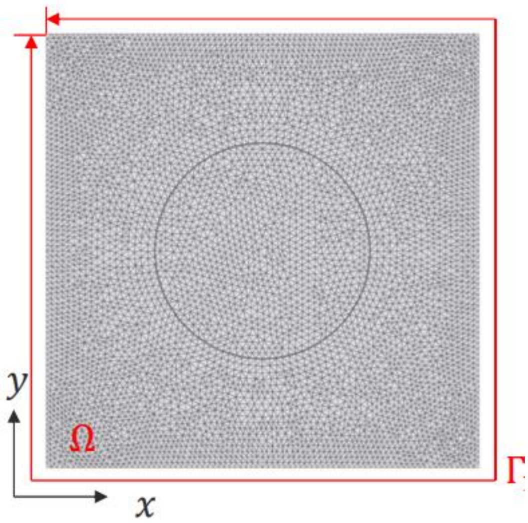
[5] There is no pressure variation in the direction of oil film thickness.

[6] There is no slip between the fluid and surface.

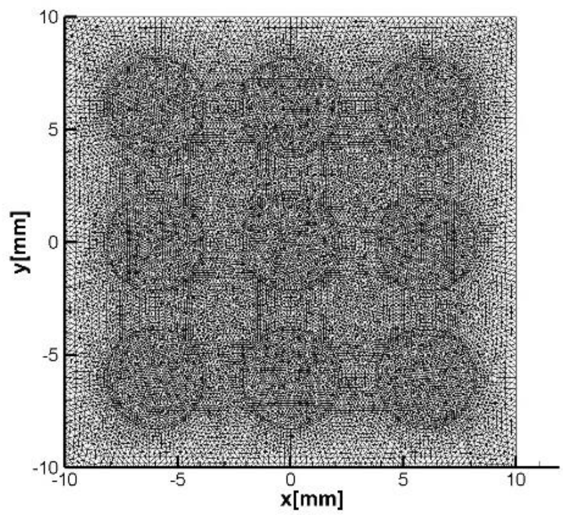
[7] The in-plane component of the rate of velocity change of the fluid is negligible compared with that of the out-of-plane component.

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\eta U \frac{\partial h}{\partial x} \text{in } \Omega \tag{2}$$

The Reynolds equation was used as a constraint function. The boundary conditions in Eq. (2) are expressed by Eq. (3). Practically, cavitation is considered to occur in the negative-pressure area of the texture; therefore, the negative pressure was set to zero, as shown in Eq. (4).



(a) Simplified calculation model



(b) Finite-element mesh

Fig. 1 Computational model

$$p = 0 \text{ on } \Gamma_1 \tag{3}$$

$$p = 0 \text{ in p} < 0 \tag{4}$$

3. Computational conditions

The Stribeck curve is shown in Fig. 2. Numerical experiments were conducted under the conditions of the reference factor to create a line showing the change in the coefficient of friction, as indicated by the blue line in Fig. 3. The vertical axis represents the friction coefficient and the horizontal axis represents the bearing constant. The lubrication areas were divided into three categories: boundary, mixed, and fluid lubrications. Because the friction coefficient was lower in the mixed lubrication area, a reference (point O) was determined in the mixed lubrication area. Next, using the factor at point O as the reference, as presented in Table 2, the larger and smaller values of the texture parameters were defined as levels 1 and 2, respectively.

The phenomenon or result that is the subject of an experiment or investigation is known to be a characteristic. In this study, the coefficient of friction was determined using the texture parameters. Hence, factors are those that affect these properties. The conditions set for these factors are called the levels. As summarized in Table 2, the texture spacing in the x-direction is defined as factor A, texture spacing in the y-direction is defined as factor B, texture depth is defined as factor C, and area ratio of texture is defined as factor D. Next, we estimated the combination of factors that reduced the coefficient of friction at point O using the design of experiments for levels 1 and 2.

Table 1 Initial condition

Load W	10
Sliding speed U	20
Viscosity η [N]	0.08
Texture spacing in x direction [mm]	6.33
Texture spacing in y direction [mm]	6.33
Initial texture depth h_{dep} [mm]	0.008
Area ratio of texture [%]	50

Table 2 Factors and Levels

Factors	Definition of Factors	Level 1	Initial level	Level 2
A	Texture spacing in x direction [mm]	7	6.33	5.67
B	Texture spacing in y direction [mm]	5.67	6.33	7
C	Texture depth [mm]	0.009	0.008	0.007
D	Area ratio of texture [%]	49	50	51

4. The Design of Experiments Method

To investigate the relationship between factors and characteristics, such as which factors affect the characteristics, as well as how to improve the characteristics by changing the factors, and what is the value of the characteristic at that time, the factors were changed and the data obtained were statistically analyzed. However, a proper analysis cannot be conducted if the data are obtained using a sloppy setup. The design of experiment is a statistical tool that provides a method for planning data collection and analyses to obtain accurate and efficient results.

From Table 2, 128 different combinations can be obtained from multiple placements. By implementing each of them, the combination that reduces the coefficient of friction the most can be determined. However, this approach is time-consuming. Contrarily, by the design of experiment method, we can choose eight out of 128 combinations and organize them efficiently in an orthogonal table, resulting in reduction in the number of times and time of analysis.

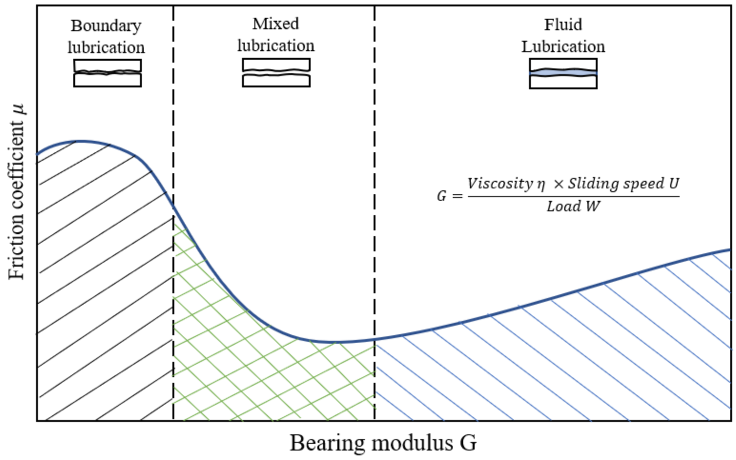


Fig. 2 The Stribeck curve

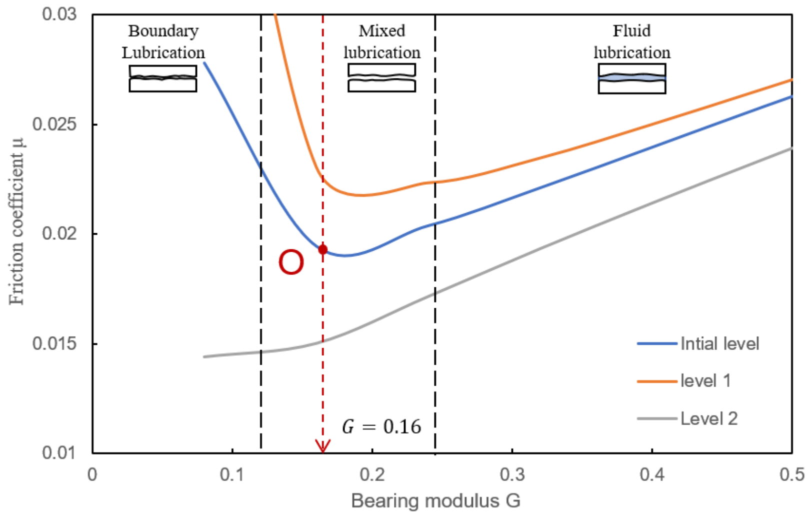


Fig. 3 Stribeck diagram from numerical experiment

Table 3 L8 orthogonal array

	B	A	E	D	A×D	B×D	C	Combination	Friction coefficient
Case 1	1	1	1	1	1	1	1	$A_1B_1C_1D_1$	0.023883
Case 2	1	1	1	2	2	2	2	$A_1B_1C_2D_2$	0.017587
Case 3	1	2	2	1	1	2	2	$A_2B_1C_1D_2$	0.012071
Case 4	1	2	2	2	2	1	1	$A_2B_1C_2D_1$	0.011518
Case 5	2	1	2	1	2	1	2	$A_1B_2C_1D_2$	0.022609
Case 6	2	1	2	2	1	2	1	$A_1B_2C_2D_1$	0.017737
Case 7	2	2	1	1	2	2	1	$A_2B_2C_1D_1$	0.013542
Case 8	2	2	1	2	1	1	2	$A_2B_2C_2D_2$	0.011952
									Summation T=0.130898

Table 4 Sum and average of each level

($T_{[k]1}$: Sum of level 1, $T_{[k]2}$: Sum of level 2, $[k]$: factors)

	B	A	E	D	A×D	B×D	C
$T_{[k]1}$	0.065058	0.081815	0.066963	0.072105	0.065643	0.069962	0.066678
Average	0.016265	0.020454	0.016741	0.018026	0.016411	0.017490	0.016670
$T_{[k]2}$	0.065840	0.049083	0.063935	0.058793	0.065255	0.060936	0.064220
Average	0.016460	0.012271	0.015984	0.014698	0.016314	0.015234	0.016055

4.1 Assign factors

In this study, there were two levels and four factors: texture spacing in x-direction (A1, A2), texture spacing in *y*-direction (B_1,B_2), texture depth (C_1,C_2), and texture area ratio (D_1,D_2). In this case, the L8 orthogonal array was appropriate. As shown in Fig. 4, we used L8 linear graphs to assign the factors. In the L8 experiment, we chose only three factors to observe all the interactions. Moreover, one column should be open to residual factor (*e*) which is the column sum of squares and represents the portion that cannot be explained by individual factors. The number of interactions from which to choose was limited to two. Factors and levels were arrayed and are presented in Table 3. The friction coefficients obtained from the eight combinations were used as evaluation values.

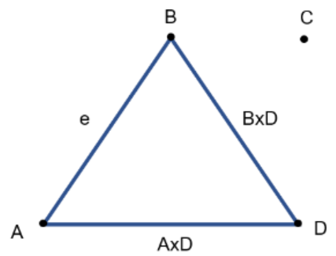


Fig. 4 L8 linear graphs

4.2 Data processing and interaction of factors

From Table 4, differences by level for each factor are calculated. Specifically, we summed and averaged the results for the friction coefficient when factor A was one (level 1), and similarly summed and averaged the results for the friction coefficient when factor A was two (level 2). In this case, we focused only on factor A and ignored all other factors. The following tables and figures present graphical representations of these sums.

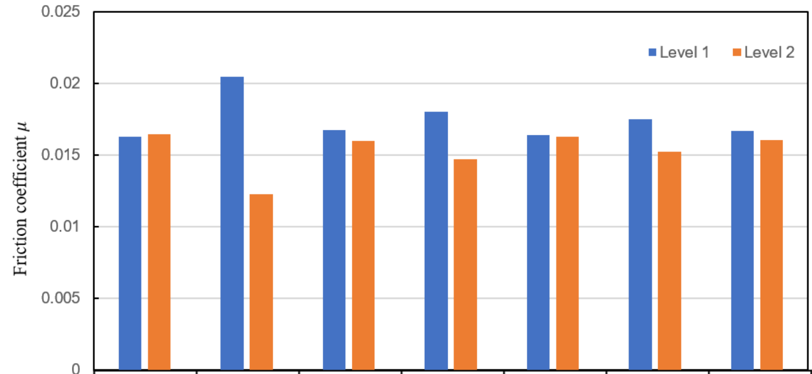
As shown in Fig. 5, the friction coefficient value of factor D at level 2 was lower than that at Level 1, whereas the friction coefficient value of factor B at Level 2 was slightly higher than that at Level 1. Basically, factors B and D are not dependent of each other but have synergistic or offsetting effects. However, factors A and D had the same high values at Level 1 and the same low values at Level 2.

Fig. 6(a) shows the duality chart between B and D, and Fig. 6(b) shows the duality chart between A and D. As shown, the two lines intersect in the top figure while they are parallel in the bottom figure. If there was no interaction between the two factors, the two lines would be parallel; however, with interaction, they would not be parallel. This is based on the fact that in the absence of interaction, each factor is independent (they do not influence each other). Basically, the interaction can be understood as the effect of one factor changing with the other.

After determining the interactions between factors, we proceeded with an analysis of variance (ANOVA) to determine whether these factors and interactions affect friction reduction.

Table 5 ANOVA table

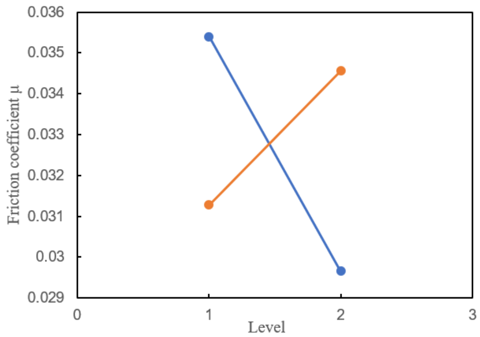
Factors	S	ϕ	V	F_0	P
A	0.00013392	1	0.00013392	116.79367	6%
B	7.6382E-08	1	7.6382E-08	0.0666143	84%
C	7.5577E-07	1	7.5577E-07	0.6591272	57%
D	2.215E-05	1	2.215E-05	19.31766	14%
A×D	1.8789E-08	1	1.8789E-08	0.0163862	92%
B×D	1.0182E-05	1	1.0182E-05	8.8799553	21%
E	1.1466E-06	1	1.1466E-06		
Total	0.00016825	7			



	B	A	e	D	A×D	B×D	C
Level 1	0.016265	0.020454	0.016741	0.018026	0.016411	0.017490	0.016670
Level 2	0.016460	0.012271	0.015984	0.014698	0.016314	0.015234	0.016055

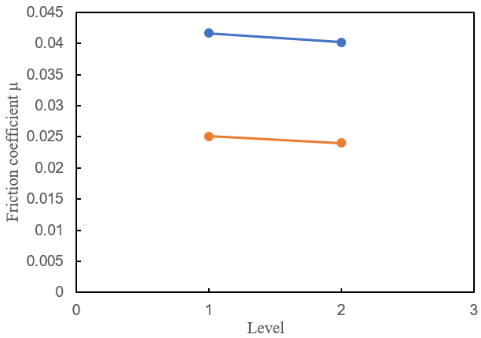
Fig. 5 Average chart of each level

	D_1	D_2
B_1	0.0354	0.029658
B_2	0.031278	0.034562



(a) For B×D

	D_1	D_2
A_1	0.041619	0.040196
A_2	0.025059	0.024024



(b) For A×D

Fig. 6 Duality table and graph

4.3 Analysis of variance (ANOVA)

Analysis of variance (ANOVA) is a statistical hypothesis-testing technique that determines the effects of factors and their interactions by decomposing the observed data variation into an error variation and variation owing to each factor and their interactions.

The sum of squares $S_{[k]}$ of the factors $[k]$ was calculated using the following equation:

$$S_{[k]} = \frac{T_{[k]1}^2}{N/2} + \frac{T_{[k]2}^2}{N/2} - \frac{T}{N} = \frac{\left(T_{[k]1} - T_{[k]2}\right)^2}{N} \quad (5)$$

where the total number of data N=8.

There were two levels for all factors; therefore, the degree of freedom for each factor was one. The overall number of degrees of freedom was seven because the number of experiments was eight.

The average square V was obtained using Eq. (6), and the equation for F_0 value is shown in Eq. (7)

$$V = \frac{S_{[k]}}{\Phi} \quad (6)$$

$$F_0 = \frac{V_{[k]}}{V_e} \quad (7)$$

Table 6 ANOVA table after pooling

Factors	S	ϕ	V	F_0	P
A	0.00013392	1	0.00013392	229.82143	0%
B	7.6382E-08	1	7.6382E-08	0.1310808	75%
C	7.5577E-07	1	7.5577E-07	2.2970014	37%
D	2.215E-05	1	2.215E-05	38.01244	3%
B×D	1.0182E-05	1	1.0182E-05	2.0936415	28%
E	1.1654E-06	2	5.8271E-07		
Total	0.00016825				

Table 5 summarizes the calculation results. The *P*-value was calculated between 0.000 and 1 and was used as a criterion to determine whether a hypothesis should be accepted or rejected. As shown in Fig. 7, a one-sided test with a significance level of 0.05 was used to determine if the results of the analysis were significant using the *P*-value. In this study, we hypothesized that these factors would significantly reduce the coefficient of friction. The hypothesis is rejected only if the *P*-value is greater than 5%, and vice versa.

From the *P*-values in Table 5, we cannot identify any factor that can be said to have a significant effect; however, this does not mean that there is no effect. This is because the orthogonal array simultaneously tests many factors. Therefore, the analysis is repeated using a pooling technique which is a method of summarizing ineffective factors into residual factor (*e*).

The guidelines for pooling are as follows:

[a] The criterion is loosened when the residual degrees of freedom are small.

[b] Pools with *P*-values greater than 20% or F_0 values less than 2

[c] When interactions are not pooled, the corresponding main effects are not pooled

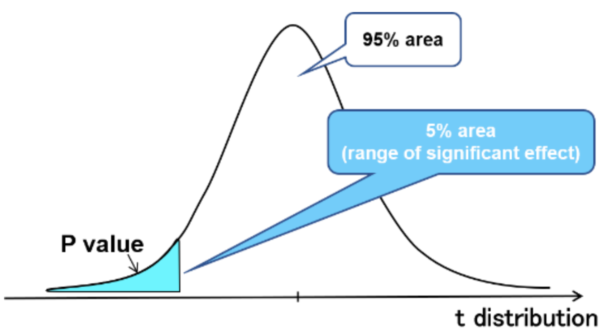


Fig. 7 One-sided test

We added factors A×D, which have the least effect, to the residual factors and create a new analysis of variance (Table 6). Analysis of variance showed that the main effects of A and D were significant while those of C and B×D, which were not significant, were not pooled because the F-value was greater than 2 [b]. Owing to [c], because the interaction factor B×D was not pooled, the corresponding main effect B was not pooled either. Consequently, after pooling, we found that factors A, B, C, D, and B×D had a significant effect on reducing the friction coefficient.

4.4 Estimation results

After determining the factors that affect the reduction in friction coefficient, it is necessary to determine the level corresponding to each factor. In this study, the smallest friction coefficient was considered; thus, the smallest value was selected. Based on Table 4, Level 2 (A_2) was chosen for factor A and Level 2 (C_2) for factor C. According to the B×D duality table, level B_1 D_2 was chosen as factor B×D. Consequently, an optimal combination of A_2 B_1 C_2 D_2, was obtained.

Eq. (8) was used to calculate the estimated friction coefficient $\hat{\mu}(\mathrm{A_2B_1C_2D_2})$ based on the selected optimal level.

$$\begin{aligned} \hat{\mu}(\mathrm{A_2B_1C_2D_2}) &= \widehat{\mu + A_2} + \widehat{\mu + C_2} \\ &\quad + \widehat{\mu + B_1 + D_2 + (BD)_{12}} \\ &= \frac{T_{[BD]12}}{2} + \frac{T_{[A]2}}{4} + \frac{T_{[C]2}}{4} - \frac{T}{2} \\ &= 0.01043025 \end{aligned} \tag{8}$$

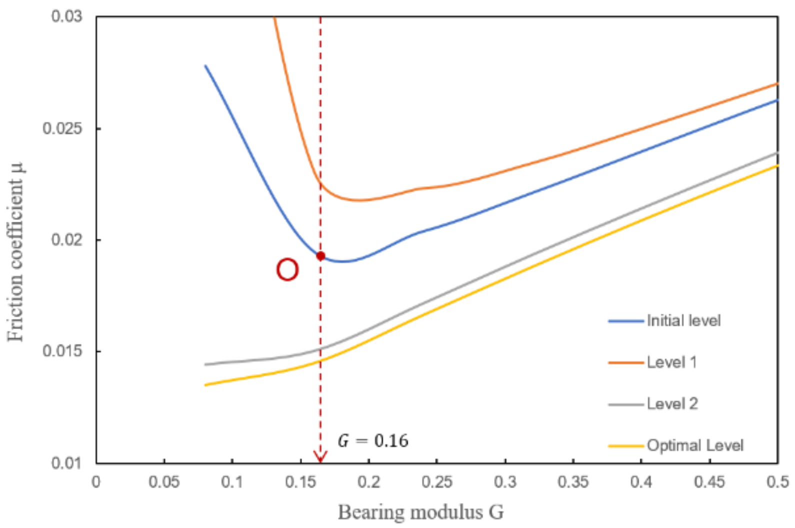


Fig. 8 Stribeck diagram from numerical experiment

5. Result

The calculated friction coefficient in FreeFEM++ with the optimal combination ($A_2B_1C_2D_2$) was 0.0112057. Compared with the results estimated by the design of experiments, the relative error was 6%, which indicates that it is a reliable value. Furthermore, by comparing the estimated and calculated values with the initial friction coefficients listed in Table 3, we found that combination A_2 B_1 C_2 D_2 had the lowest friction coefficient. In this study, texture spacing in the *x*-direction at Level 2, texture spacing in the *y*-direction at Level 1, texture depth at Level 2, and the area ratio of the texture at Level 2 are the optimal combinations.

Under the optimum combination conditions, a line showing the change in the friction coefficient was created, as indicated by the yellow line in Fig. 8. It was confirmed that the friction coefficient at point O was reduced by the optimal combination from the design of experiment method.

6. Conclusions

In this study, the design of experiments was used to determine the optimal combination of design parameters for texture analysis to reduce the coefficient of friction. The design of experiments method reduced the number of experimental analyses by approximately 94%. The number of experiments was reduced, which, in turn, reduced the time required for the study. Consequently, this study demonstrated the application of design of experiment method to evaluate the friction coefficient of textures.

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