Paper

Kinked Cracks at the Interface of Two Bonded Anisotropic Elastic Media under Antiplane Shear
-Generalized Stress Intensity Factors at the Kinked Crack Vertex-

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The kinked crack, which is reflecting back into the first medium at the interface of two bonded anisotropic media under antiplane shear loading, is analyzed by using the singular point method of complex potential functions. The crack is modeled by means of continuous distributions of dislocations, and a system of singular integral equations with generalized Cauchy kernels is obtained. The vertex of the kinked crack has the different stress singularity from that of the tips of the kinked crack. Taking account of this stress singularity of the vertex as well as the stress singularity of the kinked crack tips, the singular integral equations are solved numerically. Numerical results of the generalized stress intensity factors of the vertex of the kinked crack are shown in figures, and the influences of anisotropy of the media, the kinked crack angle, and the ratio of crack length of the kinked crack are clarified.

Key Words: *kinked crack, generalized stress intensity factors, longitudinal shear, fracture mechanics*

1. Introduction

Kinked cracks or deflected cracks are frequently observed in the fracture of brittle materials under non-uniform loadings or non-homogeneity of material properties. It is important in achievement of high fracture toughness in many brittle materials, in particular composite materials, to make clear the fracture behavior of kinked cracks at the interface of these materials. Primary concerns of this subject are to calculate the stress intensity factors at the crack tips and to clarify the behavior of crack propagation of the kinked cracks.

A number of papers in this area has been published in isotropic materials[1)~14)] so far, where fracture parameters and fracture criteria were studied. However, much less work has been done on the area for anisotropic materials[15)~19)], particularly bonded anisotropic materials. In the previous papers[20)~23)], the author has presented the analysis method of cracks and analyzed bonded anisotropic media with boundary cracks, taking account of the stress singularity of the vertex of the kinked crack; a crack terminating at the interface in arbitrary crack angle [20)], a crack kinked at and going through the interface [21)], a crack kinked at the interface and going along the interface[22)], a crack kinked at the interface back into the first medium[23)], and clarified the influence of the various factors on the stress intensity factors at the kinked crack tips.

In the present paper, we deal with the singular stress fields at the vertex of the kinked crack back into the first medium as shown in Fig.1. This problem is closely related to, for example, the design of the interface between fiber and matrix in fiber reinforced composites where it is desired that any matrix crack approaching a fiber kinks at the interface, thereby allowing the fiber survive. The solution method of this paper are founded on the previous paper[23]), based on two dimensional, anisotropic elasticity of complex potential functions. By using the singular point method, the problem is reduced to solving a system of singular integral equations with generalized Cauchy kernels. The singular stress fields near the vertex of the kinked crack are obtained by the function theoretic method[24]). Solving the singular integral equation numerically, the generalized stress intensity factor at the kinked vertex is calculated and clarified the influences of anisotropy of bonded media, kinked angles, and the ratio of the kinked crack lengths.

2. Statement of the problem and basic equations

As shown in Fig.1, a crack kinks at the interface and goes back to the first medium with arbitrary angles of α_1, α_2. The medium is made of anisotropic two semi-infinite spaces I and II with different elastic constants. These semi-infinite spaces are perfectly bonded together along the common surface L. A rectangular coordinate system (x, y) is located on the bonded surface of the media. The subsidiary rectangular coordinate systems $(x_j, y_j)(j=1,2)$ are also located on the crack L_1 of length $2a_1$, and the crack L_2 of length $2a_2$ as shown in Fig.1. The cracks $L_j(j=1,2)$ make angles $\alpha_j(j=1,2)$ with the x axis, as shown in Fig.1. The distances between the center of the crack $L_j(j=1,2)$ and the interface are $h_j(j=1,2)$. The media are subjected to antiplane shear stresses $\tau_j(j=0,1,2)$ at infinity.

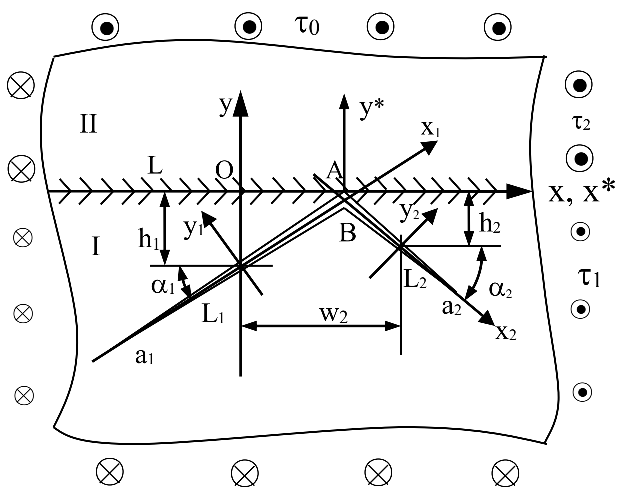

Fig.1 A crack kinked at the interface of bonded anisotropic media under longitudinal shear loadings and subsidiary coordinate systems.

The three complex variables referred to the coordinates (x, y) and $(x_j, y_j)(j=1,2)$ are defined as $z=x+iy$, $z_j=x_j+iy_j(j=1,2)$. In the following analysis, we employ the subscript j (=1, 2) for the quantities referred to the coordinate systems $(x_j, y_j)(j=1,2)$, unless stated otherwise. The Greek numerals I and II are used to denote the quantities associated with the lower and upper half-spaces, respectively.

The stress components $\sigma_{zx}^m, \sigma_{zy}^m$ and the displacement u^m (m=I , II) with respect to the rectangular coordinate system (x,y) are expressed by the two complex potential functions φ_m (m=I,II) as follows[23]):

$$\sigma_{zx}^m + i\sigma_{zy}^m = k_1^m/(1+\gamma_m)\varphi'_m(\zeta_m) + \bar{k}_2^m/(1+\bar{\gamma}_m)\overline{\varphi'_m(\zeta_m)} \tag{1}$$

$$\partial u^m/\partial x = \phi'_m(\zeta_m) + \overline{\phi'_m(\zeta_m)} \tag{2}$$

where

$$k_1^m = k_{11}^m + \gamma_m k_{12}^m,\ k_2^m = k_{12}^m + \gamma_m k_{11}^m \tag{3}$$

$$k_{11}^m = C_{55}^m - C_{44}^m + 2iC_{45}^m = \bar{k}_{22}^m,\ k_{12}^m = C_{55}^m + C_{44}^m \tag{4}$$

and C_{st} (s,t=4,5) are elastic constants for antiplane shear loading which is characterized by the symmetry with respect to the x, y plane. The characteristic value of γ_m which satisfies the condition $|\gamma_m|<1$ is obtained by the quadratic equation:

$$k_{22}^m\gamma_m^2 + 2k_{12}^m\gamma_m + k_{11}^m = 0 \tag{5}$$

The relation between the complex potential $\Phi_{mj}(\eta_m)$ with respect to the coordinate systems (x_j, y_j) in the lower half space and the complex potential $\varphi_{mj}(\zeta_m)$ are, if we write down only the potentials in the lower half space, as follows:

$$\Phi_{I1}'(\eta_1)=e^{i\alpha_1}\frac{(1+\Gamma_1)}{(1+\gamma_1)}\varphi_{I1}'(\zeta_1), \quad (6)$$

$$\Phi_{I1}'(\eta_2)=e^{-i\alpha_2}\frac{(1+\Gamma_{II})}{(1+\gamma_I)}\varphi_{I1}'(\zeta_I), \quad (7)$$

where the relation between ζ_m m=I,II) and η_j (j=1,2) is

$$\zeta_m - d_j^m = \eta_j e^{i\alpha_j}\frac{(1+\Gamma_j)}{(1+\gamma_m)} \quad (8)$$

and

$$\zeta_m = \frac{z+\gamma_m\bar{z}}{1+\gamma_m} \quad (9)$$

$$\eta_j = \frac{z_j+\Gamma_j\bar{z}_j}{1+\Gamma_j} \quad (10)$$

$$d_1^m = -ih_1\frac{(1-\gamma_m)}{(1+\gamma_m)} \quad (11)$$

$$d_2^m = w_2 - ih_2\frac{(1-\gamma_m)}{(1+\gamma_m)} \quad (12)$$

$$\Gamma_j = \gamma_m e^{-2i\alpha_j} \quad (13)$$

The constants k_j^m, which concern with the rectangular coordinates (x,y), are connected with K_j^m related to the rectangular coordinates (x_j,y_j) (j=1,2) as

$$k_1^m = K_1^m e^{2i\alpha_1},\quad k_2^m = K_2^m \quad (14)$$

The second equation shows invariant with rotation of the coordinate. The boundary conditions of this problem can be written in the forms:

(a)Along the interface of the media (y=0)

$$\sigma_{zy}^{I} = \sigma_{zy}^{II},\quad u^{I} = u^{II} \quad (15)$$

(b)Along the cracks L_j (y_j=0, $|x_j|<a_j$, j=1,2)

$$\sigma_{z_j,y_j}^{I} = 0 \quad (16)$$

(c)At infinity $(|\zeta_m| \to \infty)$

$$\sigma_{zx}^{I} = \tau_1,\quad \sigma_{zx}^{II} = \tau_2,\quad \sigma_{zy}^{I} = \sigma_{zy}^{II} = \tau_0 \quad (17)$$

3. Derivation of singular integral equations with generalized Cauchy kernels

To apply the method of continuous distributions of the screw dislocations along the kinked crack, it is necessary to obtain the fundamental complex potentials of two screw dislocations existing at the two points (x_{s0}*, y_{s0}*) (s=1,2) in the lower half space of the media. This potentials, which satisfies the boundary conditions given by Eq.(15), can be obtained by the use of the "Riemann-Hilbert boundary value problem" as follows:

$$\varphi_I'(\zeta_I) = \frac{A_1}{2\pi i}\frac{b_1}{\zeta_I-\zeta_{I0}} - \frac{A_2}{2\pi i}\frac{b_1}{\zeta_I-\bar{\zeta}_{I0}} + \frac{A_1}{2\pi i}\frac{b_2}{\zeta_I-\zeta_{II0}} - \frac{A_2}{2\pi i}\frac{b_2}{\zeta_I-\bar{\zeta}_{II0}}$$

$$\varphi_I'(\zeta_I) = \frac{A_3}{2\pi i}\left\{\frac{b_1}{\zeta_{II}-\zeta_{I0}} - \frac{b_2}{\zeta_{II}-\bar{\zeta}_{II0}}\right\} \quad (18),(19)$$

where

$$\left.\begin{aligned}&\zeta_{m0} = (z_{s0}^* + \gamma_m \bar{z}_{s0}^*)/(1+\gamma_m)\\ &z_{s0}^* = x_{s0}^* + iy_{s0}^* \quad (s=1,2)\end{aligned}\right\} \quad (20)$$

and b_j(j=1,2) are the magnitude of the Burgers vectors of the screw dislocations, and A_j(j=1,2,3) are constants including the elastic constants and given by Appendix A, and we assumed orthotropic elastic media in the present analysis, and so forth. In this case, it should be noted that the material constants k_m, k_j^m and γ_m reduce to be real.

By using the fundamental complex potentials of Eqs.(18) and (19), and distributing the dislocations on the two cracks L_1 and L_2, we assume the required complex potentials in the forms:

$$\begin{aligned}\Phi'_{I1}(\eta_1) = T_1^* &+ \frac{1}{4\pi i}\int_{-a_1}^{a_1}\frac{b_1(s_1)}{\eta_1-s_1}ds_1 + \\ &+\frac{B_1}{4\pi i}\int_{-a_1}^{a_1}\frac{b_1(s_1)}{s_1-a_1-B_2(\eta_1-a_1)}ds_1 + \\ &-\frac{B_3}{4\pi i}\int_{-a_2}^{a_2}\frac{b_2(s_2)}{s_2+a_2-B_4(\eta_1-a_1)}ds_2 + \\ &+\frac{B_5}{4\pi i}\int_{-a_2}^{a_2}\frac{b_2(s_2)}{s_2+a_2-B_6(\eta_1-a_1)}ds_2\end{aligned} \quad (21)$$

$$\begin{aligned}\Phi'_{I2}(\eta_2) = T_2^* &+ \frac{1}{4\pi i}\int_{-a_2}^{a_2}\frac{b_2(s_2)}{\eta_2-s_2}ds_2 - \\ &-\frac{C_1}{4\pi i}\int_{-a_1}^{a_1}\frac{b_1(s_1)}{s_1-a_1-C_2(\eta_2+a_2)}ds_1 + \\ &+\frac{C_3}{4\pi i}\int_{-a_1}^{a_1}\frac{b_1(s_1)}{s_1-a_1-C_4(\eta_2+a_2)}ds_1 + \\ &+\frac{C_5}{4\pi i}\int_{-a_2}^{a_2}\frac{b_2(s_2)}{s_2+a_2-C_6(\eta_2+a_2)}ds_2\end{aligned} \quad (22)$$

where B_j (j=1,…,6) and C_j (j=1,…,6) are constants and given in Appendix B, and T_1^* and T_2^* corresponding to far stress fields of the present media are given by

$$T_1^* = \tau_1 e^{i\alpha_1}D_1 + i\tau_0 e^{i\alpha_1}D_2 \quad (23)$$

$$T_2^* = \tau_1 e^{-i\alpha_2}D_1 + i\tau_0 e^{-i\alpha_2}D_2 \quad (24)$$

where D_j (j=1,2) are the constants and given by Appendix C. In derivation of Eqs. (21) and (22), Eqs. (8), (11), (12) and transformation formulas Eqs. (6) and (7) of complex potentials are used, in addition the conditions of continuity of two cracks and geometry of crack configuration are also used, namely:

$$w_2 = a_1\cos\alpha_1 + a_2\cos\alpha_2 \quad h_j = a_j\sin\alpha_j \ (j=1,2) \tag{25}$$

$$z_{10}^* = s_1 e^{i\alpha_1} - ih_1,\ \ z_{20}^* = s_2 e^{i\alpha_2} + w_2 - ih_2, \tag{26}$$

where s_j (j=1,2) denote the points on the crack L_i(i=1,2). A relation between the far field stresses τ_j(j=1,2) is given by

$$\tau_1(1+\gamma_1)/(k_1^I + k_2^I) = \tau_2(1+\gamma_{II})/(k_1^{II} + k_2^{II}) \tag{27}$$

Equation (27) can be derived from the condition of the continuity of displacement at the bonding interface similar to the case of plane problem of dissimilar media. The complex potentials Eqs.(21) and (22) of the present problem satisfy the boundary conditions of Eqs.(15) and (17) automatically. Then, to satisfy the remaining boundary condition Eq.(16), a system of singular integral equations with generalized Cauchy kernels for the density functions $b_j(s_j)$ must hold as follows :

$$\frac{1}{\pi}\int_{-1}^{1}\frac{B_1(S_1)}{S_1 - X_1}dS_1 + \frac{1}{\pi}\int_{-1}^{1}M_{11}(X_1,S_1)B_1(S_1)dS_1 + \frac{1}{\pi}\int_{-1}^{1}M_{12}(X_1,S_2)B_2(S_2)dS_2 + \frac{1}{\pi}\int_{-1}^{1}M_{13}(X_1,S_2)B_2(S_2)dS_2 = C_1^* \tag{28}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{B_2(S_2)}{S_2 - X_2}dS_2 + \frac{1}{\pi}\int_{-1}^{1}M_{21}(X_2,S_2)B_2(S_2)dS_2 + \frac{1}{\pi}\int_{-1}^{1}M_{22}(X_2,S_1)B_1(S_1)dS_1 + \frac{1}{\pi}\int_{-1}^{1}M_{23}(X_2,S_1)B_1(S_1)dS_1 = C_2^* \tag{29}$$

where the kernels M_{ij} (X, S) and the constants C_i* (i=1, 2) are given in Appendix D. In the integral equations (28) and (29), the following non-dimensional quantities are introduced:

$$X_j = x_j/a_j\ ,\ S_j = s_j/a_j\ ,\ R = a_2/a_1\ ,\quad B_j(S_j) = b_j(s_j)\ ,\ \ (j=1,2) \tag{30}$$

The single-valuedness of the displacement yields:

$$\int_{-1}^{1}B_1(S_1)dS_1 + R\int_{-1}^{1}B_2(S_2)dS_2 = 0 \tag{31}$$

4. The characteristic equation at the vertex of the kinked crack

To obtain the characteristic equation for determination of the stress singularity of the kinked vertex, we use the function-theoretic method[24]. Taking account of the known order of stress singularity of the crack tips -1/2 and the unknown order of stress singularity ω at the vertex of the kinked crack, we assume the density functions $B_j(S_j)$(j=1,2) as follows:

$$B_1(S_1) = G_1(S_1)(1-S_1)^{\omega}(1+S_1)^{-1/2} = G_1(S_1)e^{-i\omega\pi}(S_1-1)^{\omega}(S_1+1)^{-1/2}$$

$$\text{for}\quad -1<\omega<0, -1\le S_1 \le 1 \tag{32}$$

$$B_2(S_2) = G_2(S_2)(1-S_2)^{-1/2}(1+S_2)^{\omega} = G_2(S_2)e^{-i\pi/2}(S_2-1)^{-1/2}(S_2+1)^{\omega}$$

$$\text{for}\quad -1<\omega<0, -1\le S_2 \le 1 \tag{33}$$

where $G_j(S_j)\ (j=1,2)$ are the bounded functions in the closed interval $|S_j|<1$. Since integral equations of Eqs.(28) and (29) have the generalized Cauchy kernels, and after introducing the Eqs.(32) and (33) into the integral Eqs. (28) and) (29), it must look into the singular behavior of the integrals at the upper and lower points $S_j = \pm 1$ (see Appendix E) . By using these singular behavior of integrals, and by the condition due to $G_j(S_j)\ (j=1,2)$ having the non-zero values, we obtain the characteristic equation for determination of the order of stress singularity ω of the kinked vertex in the form of Eq. (34) (see next page). Equation (34) shows that the stress singularity ω depends on the elastic constants as well as the kinked angles $\alpha_j\ (j=1,2)$. It should be noted that Eq. (34) has multiple solutions of order ω, which correspond to those of two bonded wedge problems at the kink A and B in Fig.1 for arbitrary angles of $\alpha_j\ (j=1,2)$, where the stresses are generally singular except for the particular cases. For the nonhomogeneous isotropic materials, Eq.(34) reduces to Eq.(35), namely, if we put $k_1^I = k_1^{II} = 0$, $k_2^I = 2G_I$, $k_2^{II} = 2G_{II}$, $k_I = G_I$, $k_{II} = G_{II}$, $\gamma_m = 0$ and $\Gamma = G_{II}/G_I$ (G is the shear elastic constant) in Eq.(34). Numerical results of Eq.(35) coincides with the past investigation[25].

Especially, when $\Gamma = 1$, namely, for the case of homogeneous isotropic elastic medium, Eq.(35) are reduced Eq.(36). Equation (36), of course, includes the order of stress singularities of the points A and B as shown in Fig.1.

$$\begin{aligned}
&\left\|\left\langle \mathrm{Re}\left[\frac{k_1^I e^{-2i\alpha_1} - k_2^I}{1+\gamma_I e^{-2i\alpha_1}}\right]\cos\omega\pi - \mathrm{Re}\left[\frac{(k_I - k_{II})(k_1^I e^{-i\alpha_1} - k_2^I e^{i\alpha_1})}{(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})} e^{-i\omega\pi}\left(\frac{e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}}{e^{-i\alpha_1} + \gamma_I e^{i\alpha_1}}\right)^{\omega}\right]\right\rangle \otimes\right. \\
&\left.\otimes\left\langle -\mathrm{Re}\left[\frac{k_1^I e^{2i\alpha_2} - k_2^I}{1+\gamma_I e^{2i\alpha_2}}\right]\cos\omega\pi + \mathrm{Re}\left[\frac{(k_I - k_{II})(k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2})}{(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})} e^{i\omega\pi}\left(\frac{e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}}{e^{i\alpha_2} + \gamma_I e^{-i\alpha_2}}\right)^{\omega}\right]\right\rangle\right\| - \qquad (34) \\
&-\left\|\left\langle -\mathrm{Re}\left[\frac{k_1^I e^{-i\alpha_1} - k_2^I e^{i\alpha_1}}{e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}}\left(\frac{e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}}{e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}}\right)^{\omega}\right] + \mathrm{Re}\left[\frac{(k_I - k_{II})(k_1^I e^{-i\alpha_1} - k_2^I e^{i\alpha_1})}{(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})}\left(\frac{e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}}{e^{i\alpha_2} + \gamma_I e^{-i\alpha_2}}\right)^{\omega}\right]\right\rangle \otimes\right. \\
&\left.\otimes\left\langle \mathrm{Re}\left[\frac{k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2}}{e^{-i\alpha_1} + \gamma_I e^{i\alpha_1}}\left(\frac{e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}}{e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}}\right)^{\omega}\right] - \mathrm{Re}\left[\frac{(k_I - k_{II})(k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2})}{(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})}\left(\frac{e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}}{e^{i\alpha_1} + \gamma_I e^{i\alpha_1}}\right)^{\omega}\right]\right\rangle\right\| = 0
\end{aligned}$$

$$\begin{aligned}
&[\cos\omega\pi - \{(1-\Gamma)/(1+\Gamma)\}\cos\{\omega\pi - 2\alpha_1(1+\omega)\}]\otimes \\
&\otimes[\cos\omega\pi - \{(1-\Gamma)/(1+\Gamma)\}\cos\{\omega\pi - 2\alpha_2(1+\omega)\}] - \\
&-[\cos(\alpha_1+\alpha_2)(1+\omega) - \{(1-\Gamma)/(1+\Gamma)\}\otimes \\
&\otimes\cos(\alpha_1-\alpha_2)(1+\omega)]^2 = 0 \qquad (35)
\end{aligned}$$

$$\cos^2\omega\pi - \cos^2(\alpha_1+\alpha_2)(1+\omega) = 0 \qquad (36)$$

5. Analysis of interface stresses and generalized stress intensity factors at the vertex of the kinked crack

First, we analyze the singular stress field at the kinked point A shown in Fig.1. This potential which is in the upper medium (II), will be obtained from the Eq.(19), taking account of the distributing the dislocations on the cracks and the far stress field as:

$$\begin{aligned}
&\varphi^{*\prime}_{II}(Z_{II}) = T_2^* + \\
&+\frac{k_I}{2\pi i(k_I+k_{II})}\frac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\int_{-1}^{1}\frac{B_1(S_1)}{S_1 - 1 - \dfrac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}Z_{II}}dS_1 + \\
&+\frac{k_I}{2\pi i(k_I+k_{II})}\frac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\int_{-1}^{1}\frac{B_2(S_2)}{S_2 + 1 - \dfrac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\dfrac{1}{R}Z_{II}}dS_2
\end{aligned} \qquad (37)$$

where the following non-dimensional notations were used:

$$Z_{II} = \frac{1}{a_1}\left(x^* + i\frac{1-\gamma_{II}}{1+\gamma_{II}}y^*\right),\ S_1 = s_1/a_1,\ S_2 = s_2/a_2,$$
$$R = a_2/a_1,\ B_j(S_j) = b_j(s_j) \qquad (38)$$

By using the asymptotic behavior of the integral (37) as $Z_{II} \to 0$ (see Appendix E), we obtain the complex potential of the upper half space, as follows:

$$\begin{aligned}
\varphi^{*\prime}_{II}(\zeta^*_{II}) = T_2^* &- \frac{k_I}{2\pi i(k_I+k_{II})}\frac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\frac{2^{-\frac{1}{2}}}{\sin\omega\pi}\otimes \\
&\otimes G_1(1)(Z_{II})^{\omega}\left(\frac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\right)^{\omega} + \\
&+\frac{k_I}{2\pi i(k_I+k_{II})}\frac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\frac{2^{-\frac{1}{2}}e^{-\pi i\omega}}{\sin\omega\pi}\otimes \\
&\otimes G_2(-1)\frac{1}{R^{\omega}}(Z_{II})^{\omega}\left(\frac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\right)^{\omega}
\end{aligned} \qquad (39)$$

where, Z_{II}, since $y^* = 0$ on the interface, is given by

$$Z_{II} = \frac{1}{a_1}\left(x^* + i\frac{1-\gamma_{II}}{1+\gamma_{II}}y^*\right) = \frac{x^*}{a_1} = X^* \qquad (40)$$

Thus, the interface stresses near the kinked vertex in the upper half space with respect to the coordinates (x*, y*) are given by

$$\sigma_{IIz^*x^*} \cong \mathrm{Re}\left\langle\frac{k_1^{II}+k_2^{II}}{1+\gamma_{II}}\left[T_2^* + \frac{i2^{-1/2}k_I(1+\gamma_I)}{2(k_I+k_{II})\sin\omega\pi}\otimes\right.\right.$$

$$\otimes \left(X^*\right)^{\omega}\left\{G_1(1)\left(\frac{1}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\right)^{\omega+1}-\right.$$

$$\left.-G_2(-1)e^{-\pi i\omega}\frac{1}{R^{\omega}}\left(\frac{1}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\right)^{\omega+1}\right\}\Bigg]\Bigg\rangle \qquad (41)$$

$$\sigma_{\mathrm{II}z^*y^*} \cong \mathrm{Re}\left\langle \frac{k_1^{\mathrm{II}}-k_2^{\mathrm{II}}}{1+\gamma_{\mathrm{II}}}\left[T_2^*+\frac{i2^{-1/2}k_I\left(1+\gamma_I\right)^{\omega+1}}{2\left(k_I+k_{\mathrm{II}}\right)\sin\omega\pi}\otimes\right.\right.$$

$$\otimes \left(X^*\right)^{\omega}\left\{G_1(1)\left(\frac{1}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\right)^{\omega+1}\right.$$

$$\left.\left.-G_2(-1)e^{-\pi i\omega}\frac{1}{R^{\omega}}\left(\frac{1}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\right)^{\omega+1}\right\}\Bigg]\right\rangle \qquad (42)$$

Next, we will proceed to obtain the expression of the generalized stress intensity factor of the kinked vertex. The generalized stress intensity factor k_3^* at the kinked point O_3 in Fig.2, is defined in regard to the direction of the coordinate x_3:

$$\lim_{r\to 0}(2r)^{-\omega}\sigma_{z_3y_3}\Big|_{\theta=0}=k_3^* \qquad (43)$$

where $\sigma_{z_3y_3}$ can be obtained from the complex potential with respect to the coordinates (x_3*,y_3*) using the coordinate transformation formula. Thus, the complex potentials with respect to the coordinates (x_3, y_3) can be written as follows:

$$\Phi_{\mathrm{II}}^{*}{}'(\eta_3)=-\frac{k_I}{2i\left(k_I+k_{\mathrm{II}}\right)}\frac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\frac{1+\Gamma_3}{1+\gamma_{\mathrm{II}}}e^{i\theta}\otimes$$

$$\otimes\frac{2^{-\frac{1}{2}}}{\sin\omega\pi}G_1(1)\left(\eta_3\right)^{\omega}\left\{\frac{\left(1+\gamma_I\right)\left(1+\Gamma_3\right)e^{i\theta}}{\left(e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}\right)\left(1+\gamma_{\mathrm{II}}\right)}\right\}^{\omega}+$$

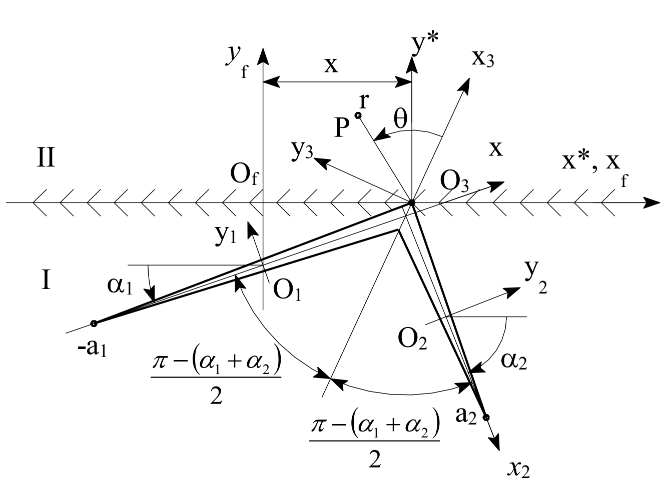

Fig.2 Coordinate systems to decide the generalized stress intensity factor at the kinked vertex.

$$+\frac{k_I}{2\pi i\left(k_I+k_{\mathrm{II}}\right)}\frac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\frac{1+\Gamma_3}{1+\gamma_{\mathrm{II}}}e^{i\theta}\,\frac{2^{-\frac{1}{2}}}{\sin\omega\pi}\otimes$$

$$\otimes G_2(-1)\left(\eta_3\right)^{\omega}e^{-\pi i\omega}\frac{1}{R}\left\{\frac{\left(1+\gamma_I\right)\left(1+\Gamma_3\right)e^{i\theta}}{\left(e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}\right)\left(1+\gamma_{\mathrm{II}}\right)}\right\}^{\omega} \qquad (44)$$

where

$$\eta_3=\left\{x_3+iy_3(1-\Gamma_3)/(1+\Gamma_3)\right\}/a_1$$

and Γ_3 is a characteristic constant with respect to coordinates (x_3,y_3). Therefore, the singular stress fields near the point O_3 are written as

$$\sigma_{\mathrm{II}z_3x_3}=\mathrm{Re}\left[\frac{ik_I\left(k_1^{\mathrm{II}}e^{-i\theta}+k_2^{\mathrm{II}}e^{i\theta}\right)2^{-\frac{1}{2}}}{2\left(k^{I}+k_{\mathrm{II}}\right)\sin\omega\pi}\cdot\frac{\left(e^{i\theta}+\gamma_{\mathrm{II}}e^{-i\theta}\right)^{\omega}\left(1+\gamma_I\right)^{\omega+1}}{\left(1+\gamma_{\mathrm{II}}\right)^{\omega+1}}\otimes\right.$$

$$\left.\otimes\left(\eta_3^*\right)^{\omega}\left\{\frac{G_1(1)}{\left(e^{i\alpha_1}+\gamma_{\mathrm{II}}e^{-i\alpha_1}\right)^{\omega+1}}-\frac{G_2(-1)e^{-\pi i\omega}}{\left(e^{-i\alpha_2}+\gamma_{\mathrm{II}}e^{i\alpha_2}\right)^{\omega+1}}\frac{1}{R^{\omega}}\right\}\right] \qquad (45)$$

$$\sigma_{\mathrm{II}z_3y_3}=\mathrm{Im}\left[\frac{ik_I\left(k_{\mathrm{II}1}e^{-i}+k_{\mathrm{II}2}e^{i}\right)2^{-\frac{1}{2}}}{2\left(k_I+k_{\mathrm{II}}\right)\sin\omega\pi}\cdot\frac{\left(e^{i}+\gamma_{f\mathrm{II}}e^{-i}\right)^{\omega}\left(1+\gamma_{fI}\right)^{\omega+1}}{\left(1+\gamma_{f\mathrm{II}}\right)^{\omega+1}}\otimes\right.$$

$$\left.\otimes\left(\eta_3^*\right)^{\omega}\left\{\frac{G_1(1)}{\left(e^{i\alpha_1}+\gamma_{\mathrm{II}}e^{-i\alpha_1}\right)^{\omega+1}}-\frac{G_2(-1)e^{-\pi i\omega}}{\left(e^{-i\alpha_2}+\gamma_{\mathrm{II}}e^{i\alpha_2}\right)^{\omega+1}}\frac{1}{R^{\omega}}\right\}\right] \qquad (46)$$

From Eqs.(43) and (46), the generalized stress intensity factor then is

$$k_3^*=\mathrm{Re}\left[\frac{k_I\left(k_1^{\mathrm{II}}e^{-i\theta}+k_2^{\mathrm{II}}e^{i\theta}\right)}{2\left(k_I+k_{\mathrm{II}}\right)\sin\omega\pi}\frac{\left(e^{1\theta}+\gamma_{\mathrm{II}}e^{-i\theta}\right)^{\omega}\left(1+\gamma_I\right)^{\omega+1}}{\left(1+\gamma_{\mathrm{II}}\right)^{\omega+1}}\otimes\right.$$

$$\left.\otimes a_1^{-\omega}\left\{\frac{G_1(1)}{\left(e^{i\alpha_1}+\gamma_{\mathrm{II}}e^{-i\alpha_1}\right)^{\omega+1}}-\frac{G_2(-1)e^{-\pi i\omega}}{\left(e^{-i\alpha_2}+\gamma_{\mathrm{II}}e^{i\alpha_2}\right)^{\omega+1}}\frac{1}{R^{\omega}}\right\}\right] \qquad (47)$$

In Eqs.(45), (46) and (47), the values of $G_1(1)$ and $G_2(-1)$ are obtained from the solution of the singular integral equations (28) and (29).

6. Numerical solution of the singular integral equations and the generalized stress intensity factors

6.1 Numerical solution of the singular integral equations.

For the numerical evaluation of the singular integral equations, we use Gauss-Jacobi type integration formula[26]:

$$\int_{-1}^{1}G(x,t)(1-t)^{\alpha}(1+t)^{\beta}dt\cong\sum_{1}^{n}W_kG(x,t_k) \qquad (48)$$

where

$$P_n^{(\alpha,\beta)}(t_k)=0\ ,\ k=1,\cdots,n \qquad (49)$$
$$-1<\mathrm{Re}(\alpha,\beta)<1$$

$P_n^{(\alpha,\beta)}(\cdot)$ are Jacobi polynomials and the weighting constants are given by

$$W_{lk}(\delta,\lambda,M)=-\frac{(2M+\delta+\lambda+2)\Gamma(M+\delta+1)}{(M+1)!\,(M+\delta+\lambda+1)\,\Gamma(M+\delta+\lambda+1)}\otimes$$
$$\otimes\frac{\Gamma(M+\lambda+1)\,2^{\delta+\lambda}}{P_{M+1}^{(\delta,\lambda)}(S_{lk})\dfrac{d}{dS_{lk}}P_{M+1}^{(\delta,\lambda)}(S_{lk})} \qquad (50)$$

where Γ(･) denotes Gamma function. Then, the integral equations (28) and (29) may now be expressed as

$$\sum_{k=1}^{n_1+1}G_1(S_{1k})\frac{W_{1k}}{\pi}\left\{\frac{1}{S_{1k}-X_{1j}}+M_{11}(X_{1j},S_{1k})\right\}+\sum_{k=1}^{n_2+1}G_2(S_{2k})\otimes$$
$$\otimes\frac{W_{2k}}{\pi}\{M_{12}(X_{1j'},S_{2k})+M_{13}(X_{1j'},S_{2k})\}=C_1^* \qquad (51)$$

$$\sum_{k=1}^{n_2+1}G_2(S_{2k})\frac{W_{2k}}{\pi}\left\{\frac{1}{S_{2k}-X_{2j}}+M_{21}(X_{2j'},S_{2k})\right\}+\sum_{k=1}^{n_1+1}G_1(S_{1k})\otimes$$
$$\otimes\frac{W_{1k}}{\pi}\{M_{22}(X_{2j},S_{2k})+M_{23}(X_{2j},S_{1k})\}=C_2^*$$
$$(j=1,2,\cdots,n_1\ :\ j'=1,2,\cdots,n_2) \qquad (52)$$

where

$$\left.\begin{aligned}
&W_{1k}(\delta,\lambda,M)=W_{1k}\left(\omega,-\frac{1}{2},n_1+1\right)\\
&P_{n_1+1}^{\left(\omega,-\frac{1}{2}\right)}(S_{1k})=0\ ,\ (k=1,2,\cdots,n_1+1)\\
&P_{n_1}^{\left(\omega+1,\frac{1}{2}\right)}(X_{1j})=0\ ,\ (j=1,2,\cdots,n_1)\\
&W_{2k}(\delta,\lambda,M)=W_{2k}\left(-\frac{1}{2},\omega,n_2+1\right)\\
&P_{n_2+1}^{\left(-\frac{1}{2},\omega\right)}(S_{2k})=0\ ,\ (k=1,2,\cdots,n_2+1)\\
&P_{n_2}^{\left(\frac{1}{2},\omega+1\right)}(X_{1j'})=0\ ,(j'=1,2,\cdots,n_2)
\end{aligned}\right\} \qquad (53)$$

On the other hand, the single-valuedness of the displacement Eq.(31) is

$$\sum_{k=1}^{n_1+1}W_{1k}G_1(S_{1k})+R\sum_{k=1}^{n_2+1}W_{2k}G_2(S_{2k})=0 \qquad (54)$$

Equations (51) and (52) include (n_1+n_2+2) unknown constants, and number of the algebraic equations is (n_1+n_2), and it is necessary more two equations. One of these is the single-valued condition Eq.(54), and the other equation is as follows:

$$\left\langle \mathrm{Re}\left[\frac{k_1^Ie^{-2i\alpha_1}-k_2^I}{1+\gamma_Ie^{-2i\alpha_1}}\right]\cos\omega\pi-\mathrm{Re}\left[\frac{(k_I-k_{II})}{(k_I+k_{II})}\otimes\right.\right.$$
$$\left.\left.\otimes\frac{(k_1^Ie^{-i\alpha_1}-k_2^Ie^{i\alpha_1})}{(e^{-i\alpha_1}+\gamma_Ie^{i\alpha_1})}e^{-i\omega\pi}\left(\frac{e^{i\alpha_1}+\gamma_Ie^{-i\alpha_1}}{e^{-i\alpha_1}+\gamma_Ie^{i\alpha_1}}\right)^{\omega}\right]\right\rangle G_1(1)+$$
$$+\left\langle -\mathrm{Re}\left[\frac{k_1^Ie^{-i\alpha_1}-k_1^{II}e^{i\alpha_1}}{e^{-i\alpha_2}+\gamma_Ie^{i\alpha_2}}\frac{1}{R^{\omega}}\left(\frac{e^{i\alpha_1}+\gamma_Ie^{-i\alpha_1}}{e^{-i\alpha_2}+\gamma_Ie^{i\alpha_2}}\right)^{\omega}\right]+\right. \qquad (55)$$
$$+\mathrm{Re}\left[\frac{(k_I-k_{II})(k_1^Ie^{-i\alpha_1}-k_1^{II}e^{i\alpha_1})}{(k_I+k_{II})(e^{i\alpha_2}+\gamma_Ie^{-i\alpha_2})}\otimes\right.$$
$$\left.\left.\otimes\frac{1}{R^{\omega}}\left(\frac{e^{i\alpha_1}+\gamma_Ie^{-i\alpha_1}}{e^{i\alpha_2}+\gamma_Ie^{-i\alpha_2}}\right)^{\omega}\right]\right\rangle G_2(-1)=0$$

Equation (55) can be obtained from the asymptotic behavior of the integral equation of Eq.(28). Equation (55) contains two new unknown constants of $G_1(1)$ and $G_2(-1)$, and the additional two equations to be determine these constants are

$$G_1(1)=\sum_{k=1}^{n_1+1}R_1(1)G_1(S_{1k}) \qquad (56)$$

$$G_2(-1)=\sum_{k=1}^{n_2+1}R_2(-1)G_2(S_{2k}) \qquad (57)$$

where

$$R_1(x)=\frac{(x-S_{11})\cdots(x-S_{1,k-1})(x-S_{1,k+1})\cdots(x-S_{1,n1+1})}{(S_{2k}-S_{1,1})\cdots(S_{2k}-S_{1,k-1})(S_{2k}-S_{1,k+1})\cdots(S_{2k}-S_{1,n1+1})}$$
$$=\prod_{\substack{k'=1\\k\neq k}}^{n_1+1}\frac{(x-S_{1k'})}{(S_{1k}-S_{1k'})} \qquad (58)$$

$$R_2(x)=\frac{(x-S_{21})\cdots(x-S_{2,k-1})(x-S_{2,k+1})\cdots(x-S_{2,n1+1})}{(S_{1k}-S_{21})\cdots(S_{2k}-S_{2,k-1})(S_{2k}-S_{2,k+1})\cdots(S_{2k}-S_{2,n_2+1})}$$
$$=\prod_{\substack{k'=1\\k\neq k}}^{n_2+1}\frac{(x-S_{2k'})}{(S_{2k}-S_{2k'})} \qquad (59)$$

These equations are well known as the Lagrange's interpolation formula. Thus, all of the unknown constants (n_1+n_2+4) will be determined by the (n_1+n_2+4) equations, and the generalized stress intensity factors at the vertex may be determined from the Eq.(47).

6.2 Non-dimensional, generalized stress intensity factors

The generalized stress intensity factor defined

Eq.(43) is normalized as, if only τ_1 acts on the media:

$$K_3^* = k_3^* / k_3' \otimes$$
$$\otimes \mathrm{Re}\left[\frac{k_I\left(k_1^{II}e^{-i\theta} + k_2^{II}e^{i\theta}\right)}{2(k_I + k_{II})\sin\omega\pi} \frac{\left(e^{i\theta} + \gamma_{II}e^{-i\theta}\right)^{\omega}(1+\gamma_I)^{\omega+1}}{(1+\gamma_{II})^{\omega+1}} \otimes \right.$$
$$\left. \otimes \left\{ \frac{G_1^*(1)}{\left(e^{i\alpha_1} + \gamma_{II}e^{-i\alpha_1}\right)^{\omega+1}} - \frac{G_2^*(-1)e^{-\pi i\omega}}{\left(e^{-i\alpha_2} + \gamma_{II}e^{i\alpha_2}\right)^{\omega+1}} \frac{1}{R^{\omega}} \right\} \right] \tag{60}$$

where

$$k_3' = \tau_1 a_1^{-\omega}\{1 + R\cos(\alpha_1 + \alpha_2)\}^{-\omega} \tag{61}$$

and corresponds to the entire crack length of projection of kinked crack length to the extension of main crack length, and

$$\left.\begin{aligned} G_1^*(1) &= \frac{G_1(1)}{\tau_1\{1 + R\cos(\alpha_1 + \alpha_2)\}^{-\omega}} \\ G_2^*(-1) &= \frac{G_2(-1)}{\tau_1\{1 + R\cos(\alpha_1 + \alpha_2)\}^{-\omega}} \end{aligned}\right\} \tag{62}$$

Also, for the case of τ_0 only, we obtain

$$K_3^{**} = k_3^* / k_3'' \otimes$$
$$\otimes \mathrm{Re}\left[\frac{k_I\left(k_1^{II}e^{-i\theta} - k_2^{II}e^{i\theta}\right)}{2(k_I + k_{II})\sin\omega\pi} \frac{\left(e^{i\theta} + \gamma_{II}e^{-i\theta}\right)^{\omega}(1+\gamma_I)^{\omega+1}}{(1+\gamma_{fII})^{\omega+1}} \otimes \right.$$
$$\left. \otimes \left\{ \frac{G_1^{**}(1)}{\left(e^{i\alpha_1} + \gamma_{II}e^{-i\alpha_1}\right)^{\omega+1}} - \frac{G_2^{**}(-1)e^{-\pi i\omega}}{\left(e^{-i\alpha_2} + \gamma_{II}e^{i\alpha_2}\right)^{\omega+1}} \frac{1}{R^{\omega}} \right\} \right] \tag{63}$$

where

$$k_3'' = \tau_0 a_1^{-\omega}\{1 + R\cos(\alpha_1 + \alpha_2)\}^{-\omega} \tag{64}$$

and

$$\left.\begin{aligned} G_1^{**}(1) &= \frac{G_1(1)}{\tau_0\{1 + R\cos(\alpha_1 + \alpha_2)\}^{-\omega}} \\ G_2^{**}(-1) &= \frac{G_2(-1)}{\tau_0\{1 + R\cos(\alpha_1 + \alpha_2)\}^{-\omega}} \end{aligned}\right\} \tag{53}$$

The values of Eqs.(62) and (65) can be obtained from Eqs.(51) and (52) which are divided by the Eqs.(61) and (64), respectively.

7. Numerical results of the generalized stress intensity factors and discussion

Numerical calculations of the generalized stress intensity factors of the kinked vertex were carried out on the bonded isotropic media as well as the bonded anisotropic media. The elastic constants and their symbols of bonded media used in the numerical calculations are shown in from Table 1 to Table 4, in which the symbol A denotes an isotropic medium, the symbols B and C orthotropic media. Table 2 and Table 4 show the combinations of those elastic constants. Note that the A/A is the case of homogeneous isotropic medium, and C/A and B/A are the cases of isotropic medium in the lower half space bonded to the orthotropic medium in the upper half space.

To check the accuracy of the numerical results, numerical calculations of the generalized stress intensity factor K_3* for various values of $n_1(=n_2)$ were performed for the case of isotropic, $\tau_0=0$, $\alpha_1=\alpha_2=45°$ and R=0.1, as shown in Fig.3. The difference between value of $n_1=50(=n_2)$ and $n_1=300(=n_2)$ is 0.1%, and we performed as $n_1=n_2=300$ for all the numerical calculations.

First, for the two bonded isotropic materials with different elastic constants, the generalized stress intensity factor K_3*, which is normalized as Eq.(60), as a function of the of crack length $R(=a_2/a_1)$, is plotted in Fig.4, for the case of $\tau_0=0, \tau_1\neq0, \tau_2\neq0$ at

Table 1 Symbols of elastic constants for isotropic media.

	C_{55}	C_{45}	C_{44}
A_1	1	0	1
A_2	2	0	2
$A_{0.5}$	0.5	0	0.5

Table 2 Symbols of bonded isotropic media.

Lower elastic constants	Upper elastic constants	Symbols
A_1	A_1	A_1/A_1
A_2	A_1	A_2/A_1
$A_{0.5}$	A_1	$A_{0.5}/A_1$

different values of the crack angles $\alpha_1(=\alpha_2)=15^\circ$, 30° and 45°. A homogeneous isotropic body is also depicted in the figure for comparison between them. The stress intensity factor $K_3{}^*$ for each given conditions is nearly constant for the range of R>0.3, and decreases rapidly (absolute values increase) as R decreases in the range of R<0.2.

Figure 5 shows the generalized stress intensity factor $K_3{}^{**}$, which is normalized as Eq.(61), for the case of $\tau_0\neq0,\tau_1=0,\tau_2=0$. The other conditions are the same as Fig.4. At the value of R=1, $K_3{}^{**}$ becomes zero due to their symmetry of kinked crack, as would be expected. The generalized stress intensity factor $K_3{}^{**}$ increases rapidly as R decreases, and the influences of elastic constants and crack angles on $K_3{}^{**}$ also increase rapidly.

Figure 6 shows the relation between $K_3{}^*$ and R in the case of anisotropic bonded materials for $\tau_0=0,\tau_1\neq0$ and $\tau_2\neq0$, at different values of the crack angles $\alpha_1(=\alpha_2)=15^\circ$, 30° and 45°, where the elastic constant in the lower half space is fixed with

Table 3 Symbols of elastic constants for anisotropic media.

	C_{55}	C_{45}	C_{44}
A	1	0	1
B	1	0	2
C	1	0	0.5

Table 4 Symbols of bonded anisotropic media.

Lower elastic constants	Upper elastic constants	Symbols
A	A	A/A
B	A	B/A
C	A	C/A

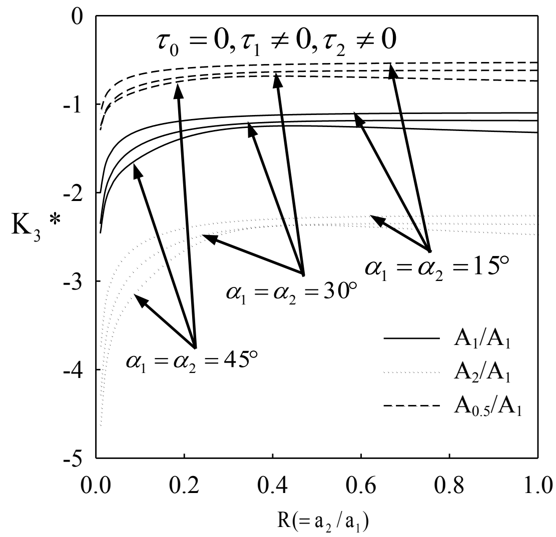

Fig.4 Effects of isotropy, $\alpha_1=\alpha_2$ and R on the stress-intensity factor $K_3{}^*$ in the case of $\tau_0=0$.

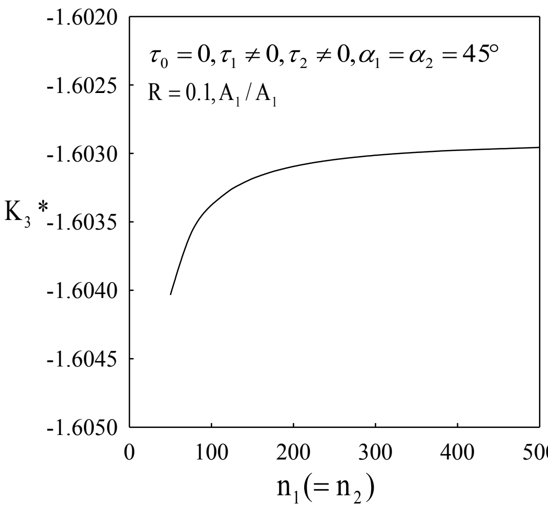

Fig.3 Effect of n_1 on the stress-intensity factor $K_3{}^*$ in the case of isotropic medium, $\tau_0=0$, and $\alpha_1=\alpha_2=45°$, R=0.1.

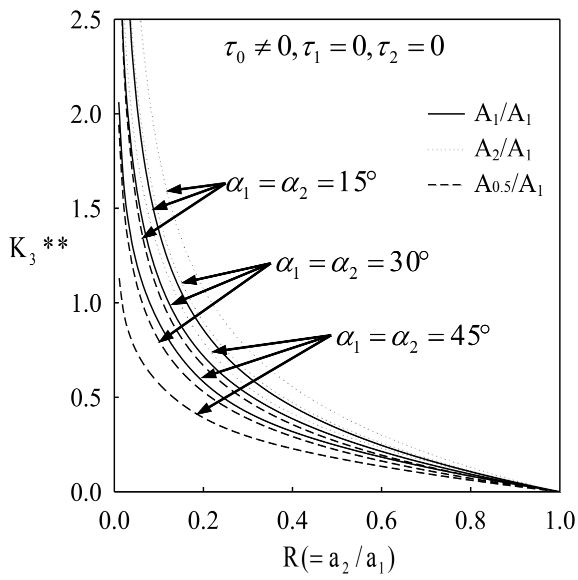

Fig.5 Effects of isotropy, $\alpha_1=\alpha_2$ and R on the stress-intensity factor $K_3{}^{**}$ in the case of $\tau_1=\tau_2=0$.

an isotropic material to make clear the influence of anisotropy. It is found from the figure that the normalized, generalized stress intensity factor K_3^* is considerably influenced by anisotropy for all the range of R.

Figure 7 shows the generalized stress intensity factor K_3^{**} , as a function of R for the case of $\tau_0 \neq 0, \tau_1=0$ and $\tau_2=0$. The other conditions are the same as the case of Fig.6. It can be seen from the figure that anisotropy of the materials has significant effects on the generalized stress intensity factor K_3^{**}, as decreasing R. This phenomenon is similar to the case of the Fig.5.

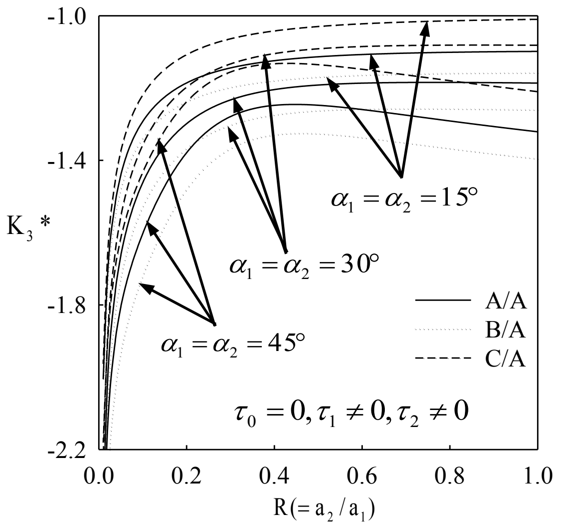

Fig.6 Effects of anisotropy, $\alpha_1=\alpha_2$ and R on the stress-intensity factor K_3^* in the case of $\tau_0=0$.

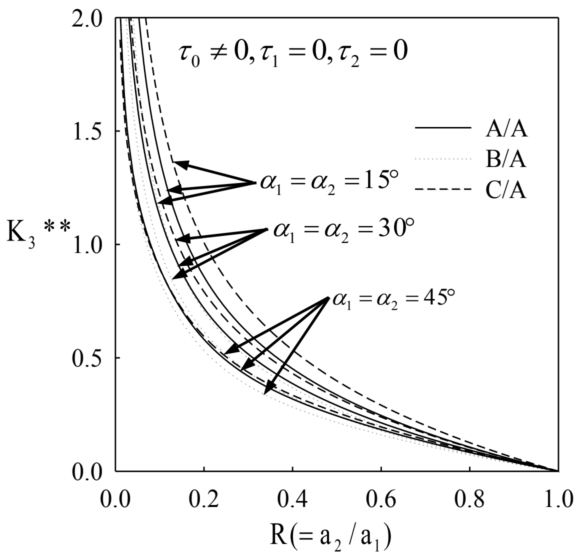

Fig.7 Effect of anisotropic, $\alpha_1=\alpha_2$ and R on the stress-intensity factor K_3^{**} in the case of $\tau_1= \tau_2 =0$.

8. Conclusion

The singular point method connected with the two dimensional anisotropic elasticity of complex potential functions has been developed for the singular stress analysis of the vertex of the kinked crack at the interface of two bonded anisotropic media under antiplane shear loadings. The problem was reduced to solving a system of the singular integral equations with generalized Cauchy kernels. The singular stress field near the vertex of the kinked crack was obtained with the generalized stress intensity factor. Solving the singular integral equations numerically, the generalized stress intensity factor at the vertex of the kinked crack was calculated, and it was found that the influences of anisotropy of the bonded media, kinked angles and the crack length ratio of the kinked crack on the generalized stress intensity factor were considerable large.

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Appendix

A

$$\left.\begin{aligned} A_1 &= \bar{k}_I / (k_I + \bar{k}_I) \\ A_2 &= k_I(\bar{k}_I - \bar{k}_{II}) / \{(k_I + \bar{k}_I)(k_I + \bar{k}_{II})\} \\ A_3 &= \bar{k}_I / (\bar{k}_I + k_{II}) \\ k_m &= -(k_I^m - k_{II}^m) / \{2(1+\gamma_m)\} \end{aligned}\right\} \quad \text{(a1)}$$

B

$$\left.\begin{aligned} B_1 &= (k_I - k_{II})(1+\Gamma_1)e^{i\alpha_1} / \{(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})\} \\ B_2 &= (e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}) / (e^{-i\alpha_1} + \gamma_I e^{i\alpha_1}) \\ B_3 &= (1+\Gamma_1)e^{i\alpha_1} / (e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}) \\ B_4 &= (e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}) / (e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}) \\ B_5 &= (k_I - k_{II})(1+\Gamma_1)e^{i\alpha_1} / \{(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})\} \\ B_6 &= (e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}) / (e^{i\alpha_2} + \gamma_I e^{-i\alpha_2}) \end{aligned}\right\} \quad \text{(a2)}$$

$$\left.\begin{aligned}
C_1 &= (1+\Gamma_2)e^{-i\alpha_2}/(e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}) \\
C_2 &(e^{-i\alpha_2}+\gamma_I e^{i\alpha_2})/(e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}) \\
C_3 &= (k_I-k_{II})(1+\Gamma_2)e^{-i\alpha_2}/\{(k_I+k_{II})(e^{-i\alpha_1}+\gamma_I e^{i\alpha_1})\} \\
C_4 &= (e^{-i\alpha_2}+\gamma_I e^{i\alpha_2})/(e^{-i\alpha_1}+\gamma_I e^{i\alpha_1}) \\
C_5 &= (k_I-k_{II})(1+\Gamma_2)e^{-i\alpha_2}/\{(k_I+k_{II})(e^{i\alpha_2}+\gamma_I e^{-i\alpha_2})\} \\
C_6 &= (e^{-i\alpha_2}+\gamma_I e^{i\alpha_2})/(e^{i\alpha_2}+\gamma_I e^{-i\alpha_2})
\end{aligned}\right\} \tag{a3}$$

C

$$\left.\begin{aligned}
D_1 &= (1+\Gamma_1)/(k_1^I+k_2^I) \\
D_2 &= (1+\Gamma_1)/(k_1^I-k_2^I)
\end{aligned}\right\} \tag{a4}$$

D

$$\left.\begin{aligned}
M_{11}(X_1,S_1) &= -E_1^{-1}\cdot \mathrm{Re}\left[F_1\{S_1-1-B_2(X_1-1)\}^{-1}\right] \\
M_{12}(X_1,S_2) &= E_1^{-1}\cdot \mathrm{Re}\left[F_2\{S_2+1-B_4(X_1-1)/R\}^{-1}\right] \\
M_{13}(X_1,S_2) &= -E_1^{-1}\cdot \mathrm{Re}\left[F_3\{S_2+1-B_6(X_1-1)/R\}^{-1}\right] \\
M_{21}(X_2,S_2) &= -E_2^{-1}\cdot \mathrm{Re}\left[F_4\{S_2+1-C_6(X_2-1)\}^{-1}\right] \\
M_{22}(X_2,S_1) &= E_2^{-1}\cdot \mathrm{Re}\left[F_5\{S_1-1-C_2R(X_2-1)\}^{-1}\right] \\
M_{23}(X_2,S_1) &= -E_2^{-1}\cdot \mathrm{Re}\left[F_6\{S_1-1-C_4(X_2-1)\}^{-1}\right]
\end{aligned}\right\} \tag{a5}$$

where

$$\left.\begin{aligned}
E_1 &= \mathrm{Re}\left[(k_1^I e^{-2i\alpha_1}-k_2^I)/2(1+\gamma_I e^{-2i\alpha_1})\right] \\
E_2 &= \mathrm{Re}\left[(k_1^I e^{2i\alpha_2}-k_2^I)/2(1+\gamma_I e^{2i\alpha_2})\right] \\
F_1 &= (k_I-k_{II})(k_1^I e^{-i\alpha_1}-k_2^I e^{i\alpha_1})/ \\
&\quad /\{2(k_I+k_{II})(e^{-i\alpha_1}+\gamma_I e^{i\alpha_1})\} \\
F_2 &= (k_1^I e^{-i\alpha_1}-k_2^I e^{i\alpha_1})/\{2(e^{-i\alpha_2}+\gamma_I e^{i\alpha_2})\} \\
F_3 &= (k_I-k_{II})(k_1^I e^{-i\alpha_1}-k_2^I e^{i\alpha_1})/ \\
&\quad /\{2(k_I+k_{II})(e^{i\alpha_2}+\gamma_I e^{-i\alpha_2})\} \\
F_4 &= (k_I-k_{II})(k_1^I e^{i\alpha_2}-k_2^I e^{-i\alpha_2})/ \\
&\quad /\{2(k_I+k_{II})(e^{i\alpha_2}+\gamma_I e^{-i\alpha_2})\} \\
F_5 &= (k_1^I e^{i\alpha_2}-k_2^I e^{-i\alpha_2})/\{2(e^{i\alpha_1}+\gamma_I e^{-i\alpha_1})\} \\
F_6 &= (k_I-k_{II})(k_1^I e^{i\alpha_2}-k_2^I e^{-i\alpha_2})/ \\
&\quad /\{2(k_I+k_{II})(e^{-i\alpha_1}+\gamma_I e^{i\alpha_1})\}
\end{aligned}\right\} \tag{a6}$$

and

$$\left.\begin{aligned}
C_1^* &= -2\left\{(k_1^I)^2-(k_2^I)^2\right\}^{-1}/E_1 \\
&\quad \times\left[\tau_1\,\mathrm{Im}\{(k_1^I-k_2^I)(k_1^I e^{-i\alpha_1}-k_2^I e^{i\alpha_1})\}+\right. \\
&\quad \left.+\tau_0\,\mathrm{Re}\{(k_1^I+k_2^I)(k_1^I e^{-i\alpha_1}-k_2^I e^{i\alpha_1})\}\right] \\
C_2^* &= -2\left\{(k_1^I)^2-(k_2^I)^2\right\}^{-1}/E_2 \\
&\quad \times\left[\tau_1\,\mathrm{Im}\{(k_1^I-k_2^I)(k_1^I e^{i\alpha_2}-k_2^I e^{-i\alpha_2})\}+\right. \\
&\quad \left.+\tau_0\,\mathrm{Re}\{(k_1^I+k_2^I)(k_1^I e^{i\alpha_2}-k_2^I e^{-i\alpha_2})\}\right]
\end{aligned}\right\} \tag{a7}$$

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For example,

$$\left.\begin{aligned}
&\frac{1}{\pi}\int_{-1}^{1}\frac{G_1(S_1)(1-S_1)^{\omega}(1+S_1)^{-\frac{1}{2}}}{S_1-X_1}dS_1 \cong 2^{-1/2}\cot\omega\pi(1-x_1)^{\omega}G_1(1) \\
&\frac{1}{\pi}\int_{-1}^{1}\frac{G_1(S_1)(1-S_1)^{\omega}(1+S_1)^{-\frac{1}{2}}}{S_1-1-\dfrac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}Z_{II}}dS_1 \cong \\
&\qquad \cong \frac{2^{-\frac{1}{2}}}{\sin\omega\pi}G_1(1)(Z_{II})^{\omega}\left(\frac{1+\gamma_I}{e^{i\alpha_1}+\gamma_I e^{-i\alpha_1}}\right)^{\omega} \\
&\frac{1}{\pi}\int_{-1}^{1}\frac{G_2(S_2)e^{-i\pi/2}(S_2-1)^{-1/2}(S_2+1)^{\omega}}{S_2+1-\dfrac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\dfrac{1}{R}Z_{fII}}dS_2 \cong \\
&\qquad \cong -\frac{2^{-\frac{1}{2}}e^{-\pi i\omega}}{\sin\omega\pi}G_2(-1)\frac{1}{R^{\omega}}(Z_{II})^{\omega}\left(\frac{1+\gamma_I}{e^{-i\alpha_2}+\gamma_I e^{i\alpha_2}}\right)^{\omega}
\end{aligned}\right\} \tag{a8}$$

The other asymptotic formulas of the Cauchy integrals are obtained similar to the Eqs.(a8), and they are omitted.

References

1) F. Erdogan and V. Biricikoglu: *Two bonded half planes with a crack going through the interface*, International Journal of Engineering Science., Vol.11, pp.745-766, 1973.
2) F. Erdogan and T. S. Cook: *Antiplane shear crack terminating at and going through a bimaterial interface*, Internationa Journal of Fracture, Vol.10, pp.227-240, 1974.
3) J. G. Goree and W. A. Venezia: *Bonded elastic half-planes with an interface crack and a perpendicular intersecting crack that extends into the adjacent material*, International Journal of Engineering Science, Vol.15, Part I, pp.1-17; Part II, pp.19-27,1977.
4) H. Hayashi and S. Nemat-Nasser: *On branched interface cracks*, ASME Journal Applied .Mechanics, Vol.48, pp.529-533, 1981.
5) M-C. Lu and F. Erdogan: *Stress intensity factors in two bonded elastic layers containing cracks perpendicular to and on the interface*, Engineering Fracture Mechanics, Vol.18, Part I, pp.491-528, 1983.
6) M-Y. He and J. W. Hutchinson: *Crack deflection at an iterface between dissimilar elastic materials*, International Journal of Solids and Structures, Vol.25, pp. 1053-1067, 1989.
7) M-Y. He and J.W. Hutchinson: *Kinking of a crack out of an interface*, ASME Journal Applied .Mechanics, Vol.56, pp.270-278, 1989.
8) K. Kasano, K. Shimoyama and H. Matsumoto: *Application of a singular integral equation method to a crack branching at an interface*, JSME International Journal, Ser.1, Vol.33, pp.431-438, 1990.
9) D. J. Mukai, R. Ballarini and G. R. Miller: *Analysis of branched interface cracks*, ASME Journal Applied Mechanics, Vol.57, pp.887-893, 1990.
10) R. Yuuki and J-Q. Xu: *Fracture criteria on kinking of a crack out of the interface in dissimilar materials*, Trans

JSME Series (A), Vol.56, pp.1945-1951, 1990.
11) S. R. Choi, K. S. Lee and Y. Y. Earmme: *Analysis of a kinked interfacial crack under out-of-plane shear*, ASME Journal Applied .Mechanics, Vol.61, pp38-44, 1994.
12) W. Lee, Y-H. Yoo and H. Shin: *Reconsideration of crack deflection at planar interfaces in layered systems*, Composites Science and Technology, Vol.64, pp.2415- 2423, 2004.
13) A. Azhdari and S. Nemat-Nasser: *Energy-release rate and crack kinking in anisotropic brittle solids*, Journal of Mechanics and Physics of Solids, Vol.44, pp.929-951, 1996.
14) M. Beghini, M. Benedetti, V. Fontanari and D. Mondelli: *Stress intensity factors of inclined kinked edge cracks: A simplified approach,* Engineering fracture mechanics, Vol.81, pp.120-129,2012.
15) M. Obata, S. Nemat-Nasser and Y. Goto: *Branched cracks in anisotropic elastic solids*, ASME Journal Applied Mechanics, Vol.56, pp.858-864, 1989.
16) A. Azhdari and S. Nemat-Nasser: *Hoop stress intensity factor and crack kinking in anisotropic brittle solids*, International Journal of Solids and Structures, Vol.33, pp929-951, 1991.
17) C. Blanco, J. M. Martinez-Esnaola and C. Atkinson: *Kinked cracks in an anisotropic elastic materials*, International Journal of fracture,Vol.93,p.87-407, 998.
18) Q-H. Qin and X. Zhang: *Crack deflection at an interface between dissimilar piezoelectric materials*, International Journal of Fracture, Vol.102, pp.355-370, 2000.
19) H. G. Beam, J. W. Lee and C. B. Cui: *Analysis of a antiplane deformation*, Journal of Mechanical Science and kinked crack in an anisotropic material under Technology, Vol.26,pp.411-419,2012.
20) T. Kondo: *Singular stresses at the tips of a crack terminating at the interface of two bonded anisotropic Media subjected to longitudinal shear loading*, Engineering Fracture Mechanics, Vol.42, pp.445-451, 1992.
21) T. Kondo and H. Sekine,H: *The crack kinking at and going through the bonded interface of two semi-infinite anisotropic media under longitudinal shear*, International Journal of Engineering Science, Vol.29, pp.797-802,1991.
22) T. Kondo and H. Sekine: *Anti-plane shear of kinked interface crack in bonded dissimilar anisotropic solid*, Theoretical and Applied Fracture Mechanics, Vol.18,pp. 273-282,1993.
23) T. Kondo, M. Kobayashi and H. Sekine: *A crack kinked at the interface of bonded anisotropic elastic media under longitudinal* shear, International Journal of the Society of Materials Engineering Resources, Vol.10, pp.158-164, 2002.
24) F.Erdogan: *Mixed boundary value problems in mechanics*, In; Mechanics Today, Edited by S. Nemat-Nasser, Oxford, 1978.
25) T.Kondo, M.Tsuji, K.Hatori and M. Kobayashi: *Stress singularities at the apex of sharp notch terminating at the interface of bonded anisotropic bodies subjected to longitudinal shear loadings*, Research Reports of Nagaoka National College of Technology, Vol.26, pp. 111-126, 1990.
26) A.H.Stroud and D.Secrest: *Gaussian Quadrature formulas*, Prentice-Hall, New York, 1966.

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